Model Derivations of Uribe (2006)
"A Fiscal Theory of Sovereign Risk"

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May 1, 2026

1 Basic Environment and the Default Rate

Consider an economy populated by a large number of identical households, the UMP:

$$\max_{\{c_t\}_{t=0}^{\infty},\{D_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(c_t)$$

s.t.

$$P_t c_t + \mathbb{E}_t r_{t+1} D_{t+1} + P_t \tau_t \leq D_t + P_t y,$$

with the non-Ponzi condition

$$\lim_{j\to\infty} \mathbb{E}_t q_{t+j} D_{t+j+1} \geq 0. \tag{1}$$

Let the Lagrange multiplier on the period-t budget constraint be $\beta^t \lambda_t / P_t$. Then the FOCs become

$$\frac{\partial \mathcal{L}}{\partial c_t} : U_c(c_t) = \lambda_t, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}} : \frac{\lambda_t}{P_t} r_{t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}}. \tag{3}$$

Let R_t^f denote the gross risk-free nominal interest rate, we have

$$(R_t^f)^{-1} = \mathbb{E}_t r_{t+1}. \tag{4}$$

The government's sequential budget constraint is given by

$$B_t = R_{t-1} B_{t-1}(1-\delta_t) - \tau_t P_t, \quad t \geq 0, \tag{5}$$

with $R_{-1}B_{-1}$ given.

In equilibrium the goods market must clear. That is $c_t = y$. By (2), the marginal utility of wealth λ_t is also constant. Then by (3), we have

$$r_{t+1} = \beta \frac{P_t}{P_{t+1}}. \tag{6}$$

Combine with (4), we obtain

$$(R_t^f)^{-1} = \beta \mathbb{E}_t \frac{P_t}{P_{t+1}}. \tag{7}$$

Because all households are assumed to be identical, in equilibrium there is no borrowing or lending among them. Thus, all asset holdings by private agents are in the form of government securities. That is,

$$D_t = R_{t-1}B_{t-1}(1-\delta_t), \quad \forall t \geq 0. \tag{8}$$

By the indifference between holding government bonds and state contingent bonds, the following Euler equation must be hold:

$$\lambda_t = \beta R_t \mathbb{E}_t \left[(1-\delta_{t+1}) \frac{P_t}{P_{t+1}} \lambda_{t+1} \right].$$

Since λ_t is constant in equilibrium, the Euler equation can be written as

$$R_t^{-1} = \beta \mathbb{E}_t \left[(1-\delta_{t+1}) \frac{P_t}{P_{t+1}} \right]. \tag{9}$$

Turning back to the non-Ponzi condition (1). By (6), q_{t+j} can be represented as

$$q_{t+j} \equiv \prod_{k=0}^{t+j} r_{k+1} = \beta^{t+j+1} \frac{P_0}{P_{t+j+1}},$$

and by (8) we have

$$D_{t+j+1} = R_{t+j}B_{t+j}(1-\delta_{t+j+1}).$$

Thus, the condition (3) becomes

$$\lim_{j\to\infty} \beta^{t+j+1} \mathbb{E}_t \left[R_{t+j}(1-\delta_{t+j+1}) \frac{B_{t+j}}{P_{t+j+1}} \right] = 0. \tag{10}$$

Now we can define the equilibrium: a rational expectations competitive equilibrium is a set of processes $\{P_t, B_t.R_t, R_t^f, \delta_t\}_{t=0}^{\infty}$ satisfying (5), (7), (9), and (10), and monetary and fiscal policies to be specified later, given $R_{-1}B_{-1}$ and the exogenous process for lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$.

A focal point of our analysis is the characterization of the equilibrium distribution of the default rate δ_t.

Firstly, multiplying both sides of (5) by $R_t(1-\delta_{t+1})$ to construct the iteration form:

$$R_tB_t(1-\delta_{t+1}) = [R_t(1-\delta_{t+1})] \cdot [R_{t-1}B_{t-1}(1-\delta_t)] - [R_t(1-\delta_{t+1})] \cdot P_t\tau_t.$$

Then iterating forward j times one can write

$$\begin{aligned} R_{t+j}B_{t+j}(1-\delta_{t+j+1}) = & \left(\prod_{h=0}^{j} R_{t+h}(1-\delta_{t+h+1}) \right) R_{t-1}B_{t-1}(1-\delta_t) \\ & - \sum_{h=0}^{j} \left(\prod_{k=h}^{j} R_{t+k}(1-\delta_{t+k+1}) \right) P_{t+h}\tau_{t+h}. \end{aligned}$$

Dividing both sides by P_{t+j} to construct the same algebra form in (2):

$$R_{t+j}\frac{B_{t+j}}{P_{t+j+1}}(1-\delta_{t+j+1}) = \left(\prod_{h=0}^{j} R_{t+h}(1-\delta_{t+h+1})\frac{P_{t+h}}{P_{t+h+1}}\right) R_{t-1}\frac{B_{t-1}}{P_t}(1-\delta_t) - \sum_{h=0}^{j}\left(\prod_{k=h}^{j} R_{t+k}(1-\delta_{t+k+1})\frac{P_{t+k}}{P_{t+k+1}}\right)\tau_{t+h}.$$

Applying $\mathbb{E}_t$ on both sides, then using (9):

$$\mathbb{E}_t\left[R_{t+j}\frac{B_{t+j}}{P_{t+j+1}}(1-\delta_{t+j+1})\right] = \beta^{-j-1}R_{t-1}\frac{B_{t-1}}{P_t}(1-\delta_t) - \sum_{h=0}^{j}\beta^{h-j-1}\mathbb{E}_t\tau_{t+h}.$$

Multiplying both sides by β^j to take the same form of (10):

$$\beta^j\mathbb{E}_t\left[R_{t+j}\frac{B_{t+j}}{P_{t+j+1}}(1-\delta_{t+j+1})\right] = \beta^{-1}R_{t-1}\frac{B_{t-1}}{P_t}(1-\delta_t) - \sum_{h=0}^{j}\beta^{h-1}\mathbb{E}_t\tau_{t+h}.$$

Taking the limit for $j\to\infty$, and using (10) one obtains

$$R_{t-1}\frac{B_{t-1}}{P_t}(1-\delta_t) = \sum_{h=0}^{\infty}\beta^h\mathbb{E}_t\tau_{t+h},$$

that is,

$$\delta_t = 1 - \frac{\sum_{h=0}^{\infty}\beta^h\mathbb{E}_t\tau_{t+h}}{R_{t-1}B_{t-1}/P_t}. \tag{11}$$

2 Delaying Default Arithmetics with Taylor Rule

Consider a monetary regime characterized by a linear feedback rule of the form

$$R_t = R^* + \alpha(\pi_t - \pi^*). \tag{12}$$

Moreover, we assume that $\alpha\beta > 1$ and $R^* = \pi^*/\beta$.

Suppose that the fiscal authority decides to delay default for $T > 0$ periods. That is, it sets

$$\delta_t = 0, \quad 0 \le t < T. \tag{13}$$

In periods $t \ge T$ the default rate is set so as to guarantee that

$$\pi_t = \pi^*, \quad t \ge T. \tag{14}$$

Before period T, since (13) is satisfied, the Euler equation (9) implies that

$$R_t = \frac{\pi_{t+1}}{\beta}, \quad t \le T-2.$$

Combining this expression with (12) we get $\pi_{t+1} = \pi^* + \alpha\beta(\pi_t - \pi^*)$ for $0 \le t \le T-2$. This expression implies the following pre-default time path for inflation by iteration:

$$\pi_t = \pi^* + (\alpha\beta)^t(\pi_0 - \pi^*), \quad 0 \le t \le T-1. \tag{15}$$

To pin down the initial inflation simply, assuming that $\tau_t = \bar{\tau}$ for all t. Then by (11) and (13), in period 0 we have

$$0 = 1 - \frac{\sum_{h=0}^{\infty} \beta^h \bar{\tau}}{R_{-1}B_{-1}/P_0},$$

which implies

$$\frac{\bar{\tau}}{1-\beta} = \frac{R_{-1}B_{-1}}{P_0},$$

then we obtain

$$\pi_0 \equiv \frac{P_0}{P_{-1}} = \frac{R_{-1}B_{-1}(1-\beta)}{P_{-1}\bar{\tau}}.$$

Now we turn to period $T-1$. The Taylor rule (12) in this period states that

$$\beta R_{T-1} = \pi^* + \alpha\beta(\pi_{T-1} - \pi^*).$$

Combining this expression with (15) yields

$$\beta R_{T-1} = \pi^* + (\alpha\beta)^T(\pi_0 - \pi^*). \tag{16}$$

Finally, in period T the stabilization policy kicks in, so $\pi_T = \pi^*$. To link the last expression (16) and the default rate δ_t, we need to consider the Euler equation.

The Euler equation (9) evaluated at time $t = T-1$ then implies that $\delta_T = 1 - \pi^*/(\beta R_{T-1})$. Combining this expression with (16) yields the following solution:

$$\delta_T = 1 - \frac{\pi^*}{\pi^* + (\alpha\beta)^T(\pi_0 - \pi^*)}. \tag{17}$$

By (15) and (17), we have that if the government delays default for T periods, then

$$\lim_{T\to\infty} \pi_{T-1} = \infty \quad \text{and} \quad \lim_{T\to\infty} \delta_T = 1.$$

3 Debt Dynamics with Price Level Targeting

The monetary regime in this section is given by

$$P_t = 1, \quad t \geq 0. \tag{18}$$

Combining (18) with (7) and (9) one writes

$$(R_t^f)^{-1} = \beta,$$

and

$$R_t^{-1} = \beta \mathbb{E}_t[1 - \delta_{t+1}]. \tag{19}$$

Setting rule (18) in (11), we the following expression for the equilibrium default rate:

$$\delta_t = 1 - \frac{\sum_{h=0}^{\infty} \beta^h \mathbb{E}_t \tau_{t+h}}{R_{t-1}B_{t-1}}. \tag{20}$$

Evaluating (20) at time $t+1$, we have

$$\delta_{t+1} = 1 - \frac{\sum_{h=0}^{\infty} \beta^h \mathbb{E}_{t+1} \tau_{t+1+h}}{R_t B_t}.$$

Taking expectations conditional on information available at time t one obtains

$$\mathbb{E}_t[1-\delta_{t+1}] = \frac{\sum_{h=1}^{\infty} \beta^h \mathbb{E}_t \tau_{t+h}}{R_t B_t}.$$

Then use (19) to eliminate $\mathbb{E}_t \delta_{t+1}$ to get

$$B_t = \sum_{h=1}^{\infty} \beta^h \mathbb{E}_t \tau_{t+h}.$$

Assume the law of motion of τ_t follows an AR(1) process of the form

$$\tau_t - \bar{\tau} = \rho(\tau_{t-1} - \bar{\tau}) + \varepsilon_t,$$

then (20) can be written as

$$\delta_t = 1 - \frac{(1-\beta)(\tau_t - \bar{\tau}) + (1-\beta\rho)\bar{\tau}}{R_{t-1} B_{t-1} (1-\beta)(1-\beta\rho)}. \tag{21}$$

Now we prove that the government is NOT able to arbitrarily fix the level of the default rate in all periods following period 0.

Assume, contrary to our contention, that $\delta_t = \bar{\delta}$, $\forall t > 0$. Then evaluating (21) at $t+1$ we have that R_t is implicitly given by

$$\bar{\delta} = 1 - \frac{(1-\beta)(\tau_{t+1} - \bar{\tau}) + (1-\beta\rho)\bar{\tau}}{R_t B_t (1-\beta)(1-\beta\rho)},$$

or the direct form

$$R_t = \frac{(1-\beta)(\tau_{t+1} - \bar{\tau}) + (1-\beta\rho)\bar{\tau}}{B_t (1-\beta)(1-\beta\rho)} \cdot (1-\bar{\delta}).$$

Since τ_{t+1} is measurable w.r.t to the information set available in period $t+1$ and B_t is measurable w.r.t to information available in t, R_t must be measurable w.r.t. to information available in $t+1$. This is a contradiction, because the government needs to announce R_t in period t without any information in the future.