Part 2: Labor and Capital Taxation with Representative-Agent

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Outline

trade-off: lower labor taxes versus efficiency

Overview

- Representative agent, with elastic labor supply.
- First recursive Koopmans utility, then separable isoelastic.
- Unrestricted government debt.
- No consumption taxes.
- Bounds on capital taxes.

Representative Agent

- Representative agent has recursive $\mathscr{V}(U_0, U_1, \dots)$, per period utilities $U_t = U(c_t, n_t)$.
  - c and n are taken to be normal goods.
  - Suppose utility satisfies recursion $\mathscr{V}_t = W(U_t, \mathscr{V}_{t+1})$.
- Along steady state with $U_t = U$ and $\mathscr{V} = \mathscr{V}(U, U, \dots)$,
  - Define $\bar{\beta}(U) = W_V(U, \mathscr{V})$ as steady state discount factor.
  - Why do we need recursive preferences?
- Agent solves

$$\max_{\{c_t, n_t\}} \mathscr{V}(c_0, n_0; c_1, n_1; \dots)$$

s.t.

$$\sum_{t=0}^{\infty} \frac{c_t - w_t n_t}{\prod_{s=0}^{t-1} R_s} = R_0 k_0 + R_0^b b_0.$$

Representative Agent

- FOCs

$$\mathscr{V}_{n,t} = w_t \mathscr{V}_{c,t},$$
$$\mathscr{V}_{c,t} = R_{t+1} \mathscr{V}_{c,t+1}.$$

- Budget constraint + FOCs $\Rightarrow$ IC constraint

$$\sum_{t=0}^{\infty} (\mathscr{V}_{c,t} c_t + \mathscr{V}_{n,t} n_t) = \mathscr{V}_{c,0} (R_0 k_0 + R_0^b b_0).$$

Technology

- Standard neoclassical technology $F(k_t, n_t)$ with RC

$$c_t + g + k_{t+1} \leq F(k_t, n_t) + (1 - \delta)k_t.$$

- After-tax return

$$R_t = (1 - \tau_t)(F_{k,t} - \delta) + 1.$$

- After-tax wage

$$w_t = (1 - \tau_t^n)F_{n,t}.$$

- Bound on capital tax

$$\tau_t \leq \bar{\tau}.$$

Planning Problem

- Planner maximizes $\mathscr{V}_0$ s.t. RC, IC, and bound on tax.
- Let $\mathscr{V}_{c,t}\Lambda_t$ be the multiplier on the time t RC. Then, FOC for capital is

$$\mathscr{V}_{c,t}\Lambda_t = \mathscr{V}_{c,t+1}\Lambda_{t+1}R^*_{t+1}.$$

Zero-Tax Result

Chamley (1986) supposes that

- A1 allocation converges to interior steady state;
- A2 bounds on capital taxes are asymptotically slack;
- A3 multiplier converges, $\Lambda_t \to \Lambda > 0$.

Then, $\tau_t \to 0$, because

$$\lim_{t\to\infty} \frac{R_{t+1}}{R^*_{t+1}} = \lim_{t\to\infty} \frac{\mathscr{V}_{C,t}/\mathscr{V}_{c,t+1}}{\mathscr{V}_{C,t}/\mathscr{V}_{c,t+1}\Lambda_{t+1}} = 1.$$

Weaker Conclusion: First Best or Zero Tax Base

- In point of fact, as the third assumption holds, one does not even need the convergence of allocation to arrive at the zero-tax conclusion.
- Thus, we only suppose the first and the second assumptions hold.
- **Result #1**: If steady state discount factor locally non-constant, $\bar{\beta}'(U) \neq 0$, then $\tau_t \to 0$ still holds, but also either
  - labor taxes $\to$ 0, or
  - private wealth $\to$ 0.
- So either the first best is achieved and hence no more capital tax is necessary, or the tax base is zero.
- Only the "knife-edge" additively separable case may give rise to steady states with zero capital tax and positive labor taxes! ($\bar{\beta}$ is a constant)

Proof

- Assume the capital tax bound is not binding eventually, say from period T onwards. We only consider the economy evaluated at time $t > T$.
- Denote $\beta_t v_t$, $\beta_t \lambda_t$, and μ as the multipliers of recursive condition, resource constraint, and IC constraint, respectively.

Proof (Cont.)

FOCs become

$$\begin{aligned}
-v_t + v_{t+1} + \mu A_{t+1} &= 0, \\
-\lambda_t + \lambda_{t+1} W_{V,t} f_{k,t+1} &= 0, \\
-v_t W_{U,t} U_{c,t} + \mu W_{U,t}(U_{c,t} + U_{cc,t} c_t + U_{nc,t} n_t) + \mu B_t U_{c,t} &= \lambda_t, \\
v_t W_{U,t} U_{n,t} - \mu W_{U,t}(U_{n,t} + U_{cn,t} c_t + U_{nn,t} n_t) - \mu B_t U_{n,t} &= \lambda_t f_{n,t},
\end{aligned}$$

where

$$\begin{aligned}
A_{t+1} &\equiv \frac{1}{\beta_{t+1}} \frac{\partial}{\partial V_{t+1}} \sum_{s=0}^{\infty} \beta_s W_{U,s}(U_{c,s} c_s + U_{n,s} n_s), \\
B_{t+1} &\equiv \frac{1}{\beta_t} \frac{\partial}{\partial U_t} \beta_s W_{U,s} \sum_{s=0}^{\infty} (U_{c,s} c_s + U_{n,s} n_s).
\end{aligned}$$

Proof (Cont.)

- Since $c_t \to c$, $k_{t+1} \to k$, and $n_t \to n$. Then U_t and V_t converge, as well as their first and second derivatives.
- $\Rightarrow$
$$a_t \to a = \frac{U_c c + U_n n}{(1-\beta)U_c R},$$
where $\beta \equiv \bar{\beta}(V)$.
- $\Rightarrow A_t \to A$ and $B_t \to B$.

Proof (Cont.)

FOCs can be written as a system for multipliers,

$$
\begin{aligned}
-v_t + v_{t+1} + \mu A &= 0, \\
-v_t + \mu\left(1 + \frac{U_{cc}c}{U_c} + \frac{U_{nc}n}{U_c}\right) + \beta\frac{B}{W_U} &= \lambda_t \frac{1}{W_U U_c}, \\
-v_t + \mu\left(1 + \frac{U_{cn}c}{U_n} + \frac{U_{nn}n}{U_n}\right) + \beta\frac{B}{W_U} &= -\lambda_t \frac{f_n}{W_U U_n}, \\
-\lambda_t + \lambda_{t+1}\beta f_k &= 0.
\end{aligned}
$$

Proof (Cont.)

- Then,

$$\tau = 1 - \frac{R^*}{R} = 1 - \beta f_k = -\frac{W_U U_c}{\lambda_{t+1}} \mu A.$$

- Either $A = 0$, or $\mu = 0$, or λ_{t+1} diverges $\Rightarrow \tau = 0$.
- Also,

$$\lambda_t \tau^n = \mu \frac{W_U U_n}{f_n} \left(\frac{U_{cc} c}{U_c} + \frac{U_{nc} n}{U_c} - \frac{U_{cn} c}{U_n} - \frac{U_{nn} n}{U_n} \right).$$

- If $a > 0$,
    - λ_t diverges $\Rightarrow \tau^n = 0$.
    - $\lambda_t \to \lambda \in \mathbb{R} \Rightarrow$ contradiction.
- Otherwise, $a = 0$.

Binding Bounds?

- Was the second assumption reasonable? i.e., does capital tax bounds not bind indefinitely?
- We move to a continuous-time model, for simple bang-bang optimal tax policies.

Planning Problem

- Assume additively separable isoelastic utility,

$$\int_0^\infty \mathrm{e}^{-\rho t} u(c_t, n_t) \mathrm{d}t,$$

with

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1+\zeta}}{1+\zeta}.$$

- Resource constraint

$$c_t + g + \dot{k}_t \leq f(k_t, n_t) - \delta k_t.$$

Planning Problem

- Planner solves

$$\max \int_0^\infty \mathrm{e}^{-\rho t} u(c_t, n_t) \mathrm{d}t$$

s.t.

$$c_t + g + \dot{k}_t \leq f(k_t, n_t) - \delta k_t,$$

$$\int_0^\infty \mathrm{e}^{-\rho t} [u'(c_t)c_t - v'(n_t)n_t] = u'(c_0)(k_0 + b_0),$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho),$$

$$r_t = (1 - \tau_t)[f_k(k_t, n_t) - \delta],$$

$$\tau_t \leq \bar{\tau}.$$

Planning Problem

- Planner solves

$$\max \int_0^\infty \mathrm{e}^{-\rho t} u(c_t, n_t) \mathrm{d}t$$

s.t.

$$c_t + g + \dot{k}_t \le f(k_t, n_t) - \delta k_t,$$

$$\int_0^\infty \mathrm{e}^{-\rho t}[u'(c_t)c_t - v'(n_t)n_t] = u'(c_0)(k_0 + b_0),$$

$$\frac{\dot{c}_t}{c_t} \ge -\frac{\rho}{\sigma}.$$

- Here, for simplicity, assume $\bar{\tau} = 1$.
- Denotes the multipliers of each condition as $\mathrm{e}^{-\rho t}\lambda_t$, μ, and $\mathrm{e}^{-\rho t}\eta_t$, respectively.

FOCs become

$$
\begin{aligned}
\Phi_v v'(n_t) - \lambda_t f_n(k_t, n_t) &= 0, \\
\eta_t c_t \frac{1}{\sigma} r_t &= 0, \\
\eta_t \frac{\rho}{\sigma} + \lambda_t - \Phi_u u'(c_t) &= \dot{\eta}_t - \rho\eta_t, \\
-r^* \lambda_t &= \dot{\lambda}_t - \rho\lambda_t,
\end{aligned}
$$

where $\Phi_v \equiv 1 + \mu(1+\zeta)$ and $\Phi_u \equiv 1 + \mu(1-\sigma)$.

Bang-Bang Property and Zero-Tax Result

- Suppose $\bar{\tau} = 1$. The FOC w.r.t. r_t can be written as

$$r_t \begin{cases} = 0 & \text{if } \eta_t < 0 \\ \in [0, \infty) & \text{if } \eta_t = 0 \end{cases},$$

since r_t must be non-negative.

- The Hamiltonian is linear (non strict convex) w.r.t. the control r_t. By Krein-Mitman, the extrema of a linear functional over a convex control set are necessarily attained at extreme points. Thus, r_t must be a bang-bang control. In other words, $\exists T$, s.t. $\tau_t = \bar{\tau}$ for $t < T$, and $\tau_t = 0$ for $t > T$.
- Chamley (1986) claimed that $T < \infty$. So $\lim_{t \to \infty} \tau_t = 0$.

Question: Why can T not be infinite?

Infinite Time and Positive Long-Run Tax

- **Result #2**: Take $\bar{\tau} = 1$ and $\sigma > 1$. Fix initial capital k_0. Then $\exists \underline{b} < \overline{b}$, s.t. if $b_0 \in [\underline{b}, \overline{b}]$ then $T = \infty$.
- For sufficiently high levels of initial debt b_0, the bounds on capital taxes bind forever!

Proof

- The FOC w.r.t. c_t (η_t):

$$\dot{\eta}_t - \rho\eta_t = \eta_t \frac{\rho}{\sigma} + \lambda_t - [1 - \mu(\sigma - 1)]u'(c_t),$$

or

$$\dot{\eta}_t = \left(1 + \frac{1}{\sigma}\right)\rho\eta_t + \lambda_t - [1 - \mu(\sigma - 1)]u'(c_t),$$

where tax bound $\tau_t = \bar{\tau}$ binds if $\eta_t < 0$.

- $T < \infty \Rightarrow \eta_t = \dot{\eta}_t = 0$, $\forall t > T \Rightarrow$ for such t,

$$\lambda_t = [1 - \mu(\sigma - 1)]u'(c_t).$$

- $\sigma > 1$ and μ sufficiently large $\Rightarrow \lambda_t \geq 0$ but RHS < 0.

Contradiction!

Proof (Cont.)

When is μ sufficiently large?

- $b_0 \Rightarrow$ government needs to tax more $\Rightarrow$ IC constraint becomes tighter $\Rightarrow \mu \uparrow$.
- In fact, as b_0 approaches highest feasible debt level $\bar{b}$, $\mu \nearrow +\infty$.

Asymptotical Dynamics

- Denote $m_1 = (1 + 1/\sigma)\rho$, $m_2(t) = \lambda(t) - [1 - \mu(\sigma - 1)]u'(c(t))$. Then the FOC becomes

$$\dot{\eta}(t) = m_1 \eta(t) + m_2(t).$$

- The general solution

$$\eta(t) = \mathrm{e}^{m_1 t} \left[\eta(0) + \int_0^t \mathrm{e}^{-m_1 s} m_2(s) \mathrm{d}s \right].$$

- TVC ensures that $\eta(0) + \int_0^t \mathrm{e}^{-m_1 s} m_2(s) \mathrm{d}s \to 0$ as $t \to \infty$, then we obtain

$$\eta(t) = -\mathrm{e}^{m_1 t} \int_t^\infty \mathrm{e}^{-m_1 s} m_2(s) \mathrm{d}s.$$

Asymptotical Dynamics

- $T = \infty \Leftrightarrow \nexists T < \infty$ s.t. $\eta(T) = 0 \Leftrightarrow$ s.t.

$$\int_t^{\infty} \mathrm{e}^{-m_1 s} m_2(s) \mathrm{d}s = 0.$$

- If such $T < \infty$, then $m_2(t) = \lambda(t) - [1 - \mu(\sigma - 1)]u'(c(t))$ **must exhibit sufficiently large oscillations,** because $\mathrm{e}^{-m_1 t} > 0$.

Converging Allocation?

- Was the first assumption reasonable?
- Or in other words, is any assumption redundant?

Planning Problem

- Use same continuous time planning problem as for Chamley

$$\max \int_0^\infty \mathrm{e}^{-\rho t} u(c_t, n_t) \mathrm{d}t$$

s.t.

$$c_t + g + \dot{k}_t \leq f(k_t, n_t) - \delta k_t,$$

$$\int_0^\infty \mathrm{e}^{-\rho t} [u'(c_t)c_t - v'(n_t)n_t] = u'(c_0)(k_0 + b_0),$$

$$\frac{\dot{c}_t}{c_t} \geq -\frac{\rho}{\sigma}.$$

- We do not assume convergence of allocation, instead assume multiplier Λ_t is bounded.

Multiplier

- Call $\mathrm{e}^{-\rho t}u'(c_t)\Lambda_t$ the multiplier on the RC.
- We can prove that
$$\frac{\dot{\Lambda}_t}{\Lambda_t} = r_t - r_t^*.$$
- $\Lambda_t \to \Lambda \Rightarrow r_t - r_t^* \to 0 \Rightarrow \tau_t \to 0$.
- This result tells that **Chamley's claim actually by postulating the convergence of Λ_t.**
- Thus, we assume the endogenous multiplier remains in a bounded interval, $\Lambda_t \in [\underline{\Lambda}, \overline{\Lambda}]$, with $0 < \underline{\Lambda} < \overline{\Lambda}$.

- Judd (1999) suggested the average capital tax goes to zero,

$$\frac{1}{t}\int_0^t (r_s - r_s^*)\mathrm{d}s \to 0.$$

- Why? Bounded condition implies

$$\frac{\ln\Lambda_0 - \ln\overline{\Lambda}}{t} \leq \frac{1}{t}\int_0^t (r_s - r_s^*)\mathrm{d}s \leq \frac{\ln\Lambda_0 - \ln\underline{\Lambda}}{t},$$

then let $t \to \infty$, and by Squeeze Theorem we obtain the result.

Question: Do this result follow from imposing the bounds of Λ_t?

Alternative Interpretation

- **Result #3**: Assuming Λ_t converges (or is bounded) is essentially assuming the result!
- $\sigma > 1$ and μ sufficiently large $\Rightarrow \Lambda_t \to 0$ by Result $\#2 \Rightarrow$ the bounded assumption is violated $\Rightarrow$ the average capital tax do not always go to zero.

Proof

- FOC for capital implies

$$\mathsf{MRS}^{\mathsf{social}}_{t,t+s} = \exp \int_0^t r^*_{t+\bar{s}} \mathrm{d}\bar{s}.$$

- Using the agent's Euler equation,

$$\mathsf{MRS}^{\mathsf{private}}_{t,t+s} = \exp \int_0^t r_{t+\bar{s}} \mathrm{d}\bar{s}.$$

- It follows that

$$\frac{\Lambda_{t+s}}{\Lambda_t} = \frac{\mathsf{MRS}^{\mathsf{social}}_{t,t+s}}{\mathsf{MRS}^{\mathsf{private}}_{t,t+s}}.$$

- Then this wedge is the only source of non-zero taxes. If Λ_t is bounded, social and private MRS values coincide and the intertemporal wedge is zero.

Intuition

- An indefinite tax on capital $\Leftrightarrow$ an ever-rising tax on consumption.
- If the MC of the distortion were increasing in T and approached infinity as $T \to \infty$ this would give a strong economic rational against indefinite taxation of capital.
- BUT we can show that the MC remains bounded, even as $T \to \infty$.
- Bounds on capital tax $\Leftrightarrow$ consumption taxes to be low initially $\Rightarrow$ makes ever-rising consumption taxes a third best.

Questions are welcome.