Part 1: Capital Taxation and Redistribution with Two-Agent

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Outline

trade-off: redistribution versus efficiency

1. Model Environment
2. Zero-Tax Result and Overturning
3. Inverse Elasticity Rule and Overturning

Overview

- Two agents: capitalists and workers
- Capitalists:
  - only capital income
  - own initial capital stock k_0
- Workers:
  - only labor income, inelastic labor supply of 1
  - consume hand-to-mouth
- Policy instruments
  - capital taxes, lump-sum transfers to workers
  - no government bonds, no consumption taxes
  - full ex-ante commitment to tax policy

Capitalists

- Resource constraint

$$c_t + C_t + g + k_{t+1} \leq f(k_t) + (1 - \delta)k_t.$$

- Capitalists' problem

$$\max_{\{C_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

s.t.

$$C_t + a_{t+1} = R_t a_t,$$
$$a_{t+1} \geq 0,$$

where R_t denotes after-tax interest rate on capital, a_t denotes wealth of capitalists.

Capitalists

- Market clearing: $a_t = k_t$.
- FOC:

$$\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t.$$

- TVC:

$$\beta^t U'(C_t)k_{t+1} \to 0.$$

- IC constraint = FOC + budget constraint + market clearing.

Government and Redistribution

- Before tax interest rate: $R_t^* = f'(k_t) + 1 - \delta$.
- Wage: $w_t = f(k_t) - f'(k_t)k_t$.
- Transfers to workers: T_t.
- Workers behavior: $c_t = w_t + T_t$.
- Government budget constraint (no debt):

$$g + T_t = (R_t^* - R_t)k_t.$$

Planning Problem

Suppose the planner only cares about the utility of workers, then she solves

$$\max \sum_{t=0}^{\infty} u(c_t)$$

s.t.

$$\begin{gathered} c_t + C_t + g + k_{t+1} \leq f(k_t) + (1-\delta)k_t, \\ \beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t, \\ \beta^t U'(C_t)k_{t+1} \to 0. \end{gathered}$$

FOCs

- Assume $U(C) = C^{1-\sigma}/(1-\sigma)$.
- Denote μ_t as the multiplier on IC constraint, $\kappa_t = k_t/C_{t-1}$, $v_t = U'(C_t)/u'(c_t)$.
- FOCs:

$$\mu_0 = 0$$

$$\mu_{t+1} = \mu_t \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} v_t} (1 - \gamma v_t).$$

$$\frac{u'(c_{t+1})}{u'(c_t)} [f'(k_{t+1}) + 1 - \delta] = \frac{1}{\beta} + v_t(\mu_{t+1} - \mu_t).$$

Zero-Tax Result

Judd (1985) studies interior steady state,

- Assume $c_t \to c > 0$, $C_t \to C > 0$, $k_{t+1} \to k > 0$, $\mu_t \to \mu \in \mathbb{R}$.
- Euler equation $\Rightarrow R_t \to R = 1/\beta$.
- Last FOC $\Rightarrow R_t^* \to R^* = 1/\beta$.
- Then $\mathscr{T} = 1 - R/R^* = 0$. **Zero Capital Tax!**

Zero-Tax Result: Counterexample

Is that universally the case?

- Non-convergence (or cycles) of allocation?
- Convergence to non-interior steady state?
- Non-convergence of multipliers?

Non-convergence to Interior Steady State

- Convergence of c_t, C_t, k_{t+1}, $\mu_t \Rightarrow \kappa_t \to \kappa > 0$, $v_t \to v > 0$.
- Second FOC $\Rightarrow \mu_{t+1} - \mu_t \to \frac{\sigma-1}{\sigma\kappa}\mu + \frac{1-\gamma v}{\beta\sigma\kappa v}$.
  - $\mu_0 = 0 \Rightarrow \mu_t \geq 0$, $\forall t$
- Last FOC $\Rightarrow R_t \to R = \frac{1}{\beta} + \frac{v(\sigma-1)}{\sigma\kappa}\mu + \frac{1}{\beta\sigma\kappa}$.
  - $\beta R = 1 \Rightarrow \mu_t \to \mu = -\frac{1}{(\sigma-1)\beta v}$.
- $\sigma > 1? \Rightarrow \mu_t \to \mu < 0$

Contradiction!

Convergence to non-interior steady state

If $\sigma > 1$,

- **Result #1**: The economy cannot be converging to any interior steady state. Hence, it the optimal allocation converges, either $k_t \to 0$, $C_t \to 0$, or $c_t \to 0$.
- We can prove that there must be $c_t \to 0$. Then $k_t \to \overline{k}$ or $k_t \to \underline{k}$, where $\overline{k}$ and $\underline{k}$ are the two roots of $k = \beta F(k)$, with $F(k) := f(k) + (1-\delta)k - g$.

Question: Does the optimal allocation converge?

Main Result: Positive Long-Run Capital Tax

Result #2: If $\sigma > 1$, any solution to the planning problem converges to $c_t \to 0$, $k_t \to \underline{k}$, $C_t \to \frac{1-\beta}{\beta}\underline{k}$, with a positive limit tax on wealth: $\mathscr{T}_t \to \bar{\mathscr{T}} > 0$. The limit tax $\bar{\mathscr{T}}$ is decreasing in spending g, with $\bar{\mathscr{T}} \to 1$ as $g \to 0$.

Proof

First, write the recursive formulation with state (k_t, C_{t-1}) and control c_t. Denote Z^* as the feasible set.

$$V(k, C\_) = \max_{c \geq 0, (k', C) \in Z^*} u(c) + \beta V(k', C)$$

s.t.

$$c + C + k' + g = f(k) + (1 - \delta)k,$$
$$\beta U'(C)(C + k') = U'(C\_)k,$$
$$c, C, k' \geq 0.$$

Proof (Cont.)

The geometry of feasible set Z^*:

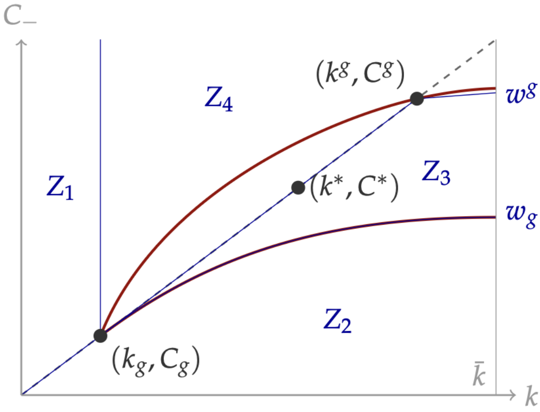

Figure: The state space of planning problem

Proof (Cont.)

We next prove that

- Step 1: μ_t explodes and $c_t \to c = 0$.
- Step 2: $c = 0 \Rightarrow$ An optimal path approaches either $(\underline{k}, \underline{C})$ or $(\overline{k}, \overline{C})$.
- Step 3: The economy cannot converge to $(\overline{k}, \overline{C})$.
- Step 4: An optimal path converges to $(\underline{k}, \underline{C})$.

Proof (Cont.)

Consider the optimal long-run tax,

- $F'(k) = f'(k) + 1 - \delta = R^*$.
- $k = \underline{k} \Rightarrow F'(k) > 1/\beta = R \Rightarrow$
  - $\mathscr{T} = 1 - R/R^* > 0$.
  - $\frac{\mathrm{d}\underline{k}}{\mathrm{d}g} = \frac{1}{F'(k) - 1/\beta} > 0 \Rightarrow \frac{\mathrm{d}\mathscr{T}}{\mathrm{d}g} < 0$.
- $g = 0 \Rightarrow \underline{k} = \beta[f(\underline{k}) + (1-\delta)\underline{k}] \Rightarrow \underline{k} = 0$ and $\mathscr{T} \to 1$.

Intuition

- Incentivizing capitalists' savings behavior through anticipatory effects
  - Start with a constant tax,
  - Announce a tax increase in the far future,
  - EIS $< 1 \Rightarrow$ capitalists increase savings today,
  - ... which is great if capital is taxed today!
- Rationalizes why the planner likes an increasing slope for capital taxes!
- Reverse holds for EIS > 1: tax decreases to zero.

Numerical Illustration

Given k_0, one can obtain the dynamic paths of $\{k_t\}$ and $\{\mathscr{T}_t\}$ by using VFI.

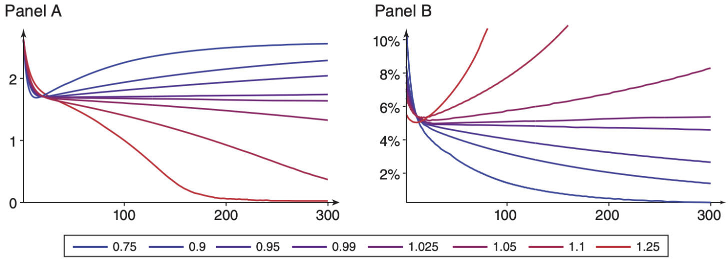

Figure: Optimal time paths for capital (panel A) and wealth taxes (panel B)

General Savings Functions

- Result #1 can be generalized to almost arbitrary savings functions for capitalists.
- Suppose having time t income I_t, capitalists save exactly $S(I_t, R_{t+1}, R_{t+2}, \dots)$,
  - It naturally depends on future interest rate $\{R_{t+1}, R_{t+2}, \dots\}$.
  - Assume S increases in I_t and weakly decreases in future interest rates.
- **Result #3**: the optimal tax rates cannot converge to zero (or anything negative).

Is an Infinite Elasticity Sufficient for Zero Long-Run Tax?

- Judd (1985) claims his zero tax result is robust to recursive utility for capitalists,
  - allows for finite long-run elasticity of savings to permanent changes in capital taxes.
- Piketty & Saez (2013) show the elasticity of the present value of savings to future tax changes is sufficient statistic for long-run tax,
  - infinite elasticity $\Leftrightarrow$ zero long-run capital tax.
- Here we can show that this is not quite right,
  - infinite elasticity $\nRightarrow$ zero long-run capital tax.

Is an Infinite Elasticity Sufficient for Zero Long-Run Tax?

- If an interior steady state is reached, the long-run wealth tax can (under regularization condition) be written as "inverse elasticity formula",

$$\mathscr{T} = 1 - \frac{R}{R^*} = \frac{1 - \beta R S_I}{1 + \sum_{t=1}^{\infty} \beta^{-t+1} \varepsilon_{S,t}}.$$

- This seems to confirm Piketty-Saez' claim: It is zero whenever $\sum_{t=1}^{\infty} \beta^{-t+1} \varepsilon_{S,t}$ is infinite. BUT we already know from Result #3 that $\mathscr{T} > 0$!
- **Result #4**: The above inverse elasticity formula is inapplicable whenever $1 + \sum_{t=1}^{\infty} \beta^{-t+1} \varepsilon_{S,t} < 0$!
    - Eg. isoelastic utility with IES < 1: infinite elasticity but $\mathscr{T} \neq 0$.

Questions are welcome.