Part 3: Labor Taxation with Heterogeneous-Agent

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Outline

trade-off: liquidity provision versus efficiency

Model Environment

Heterogeneous-agent with idiosyncratic productivity risk

$$\max_{\{c_{i,t},n_{i,t},k_{i,t+1}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{i,t}, n_{i,t}) \right]$$

s.t.

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + (1 - \tau_t)e_{i,t}n_{i,t},$$

and $a_{i,t} \geq 0$.

Model Environment

- Representative firm: $Y_t = N_t$, pre-tax wage = 1.
- Government spending is fixed, $G > 0$.
  - controls labor taxes and debt,
  - budget constraint

$$G + (1 + r_t)B_{t-1} = B_t + \tau_t N_t.$$

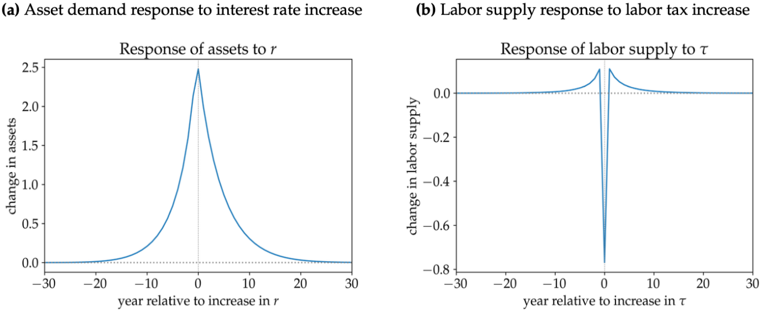
Aggregation of Household Behaviors

Given $\{r_t\}$ and $\{\tau_t\}$, one can aggregate household behavior using sequence-space functions:

$$
\begin{aligned}
A_t &= \mathscr{A}_t(\{r_s, w_s\}) \equiv \int a_t^*(e, a_-)\mathrm{d}\Psi_t(e, a_-),\\
N_t &= \mathscr{N}_t(\{r_s, w_s\}) \equiv \int n_t^*(e, a_-)\mathrm{d}\Psi_t(e, a_-),\\
C_t &= \mathscr{C}_t(\{r_s, w_s\}) \equiv \int c_t^*(e, a_-)\mathrm{d}\Psi_t(e, a_-),\\
U_t &= \mathscr{U}_t(\{r_s, w_s\}) \equiv \int u[c_t^*(e, a_-), n_t^*(e, a_-)]\mathrm{d}\Psi_t(e, a_-).
\end{aligned}
$$

Infinitely Anticipated Shocks

Consider anticipated one-time shock at some far-out future date s.



Infinitely Anticipated Shocks

- If the date $t = s + h$ response, in percentage points of steady state assets, is given by the derivative (semi-elasticity) $\partial \ln \mathscr{A}_{s+h} / \partial r_s$, where h runs from $-s$ to ∞.
- For s sufficiently large, these derivatives become independent of s, and only depend on the time horizon h relative to the date of the anticipated shock.
- Also, the derivatives (elasticities) of labor supply to an increase in labor income taxes (equivalent to a decrease in after-tax wages) can be written as $-\partial \ln \mathscr{N}_{s+h} / \partial \ln w_s$, with a large date s.

Question: How to summarize the responses to infty anticipated shocks?

Discounted Elasticities

- Define useful "discounted" version of these derivatives:

$$\epsilon^{A,r}(\delta) \equiv \lim_{s\to\infty} \sum_{h=-s}^{\infty} \delta^h \frac{\partial \ln \mathscr{A}_{s+h}}{\partial r_s},$$
$$\epsilon^{N,\tau}(\delta) \equiv \lim_{s\to\infty} \sum_{h=-s}^{\infty} \delta^h \frac{\partial \ln \mathscr{N}_{s+h}}{\partial \tau_s/(1-\tau)},$$

  and all the other elasticities similarly.
- These elasticities are discounted with some δ (later social discounted factor.
- Generalize similar elasticities in Piketty-Saez and Straub-Werning.

Competitive Equilibrium

The prices $\{r_t, w_t\}$ $(w_t = 1 - \tau_t)$ are part of a competitive equilibrium $\Leftrightarrow$

$$\mathscr{C}_t(\{r_s, w_s\}) + G = \mathscr{N}_t(\{r_s, w_s\}),$$

$\Leftrightarrow$

$$G + (1 + r_t)\mathscr{A}_{t-1}(\{r_s, w_s\}) = \mathscr{A}_t(\{r_s, w_s\}) + (1 - w_t)\mathscr{N}_t(\{r_s, w_s\}).$$

Ramsey Problem

- Full-commitment problem, with arbitrary social discount factor δ,

$$\max_{\{r_s, w_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathscr{U}_t(\{r_s, w_s\})$$

s.t.

$$G + (1 + r_t)\mathscr{A}_{t-1}(\{r_s, w_s\}) = \mathscr{A}_t(\{r_s, w_s\}) + (1 - w_t)\mathscr{N}_t(\{r_s, w_s\}).$$

- If solution converges to well-defined steady state, $r_s \to r$, we call this steady state a Ramsey steady state (RSS).
- Denote the multiplier on the constraint as $\delta^t \lambda_t$.

Diverge Multiplier

- Do λ_t always converge? NO!
- If $\lambda_t \to \infty$ with a asymptotical constant growth rate

$$\lambda_t/\lambda_{t-1} \to \eta \in [1, 1/\delta],$$

then in RSS, the constraint and as well as two optimality conditions below have to hold:

$$(1-\delta\eta(1+r))\ell\epsilon^{A,\tau}(\delta\eta) - \frac{1-w}{w}(-\epsilon^{N,\tau}(\delta\eta)) + 1 = 0,$$
$$(1-\delta\eta(1+r))\ell\epsilon^{A,r}(\delta\eta) - \frac{1-w}{w}(-\epsilon^{N,r}(\delta\eta)) - \ell = 0,$$

where $\ell \equiv A/(wN)$ denotes a unit-less measure of liquidity in RSS.

Begin with the FOCs w.r.t. r_s:

$$\overbrace{\sum_{h=-s}^{\infty} \delta^h \frac{\partial \mathscr{U}_{s+h}}{\partial r_s}}^{\epsilon^{U,r} \text{ as } s \to \infty} + \sum_{h=-s}^{\infty} \delta^h \lambda_{s+h} \left(\overbrace{\frac{\partial \mathscr{A}_{s+h}}{\partial r_s}}^{A\lambda \cdot \epsilon^{A,r}} + \tau_t \overbrace{\frac{\partial \mathscr{N}_{s+h}}{\partial r_s}}^{\tau N \lambda \cdot \epsilon^{N,r}} - (1 + r_t) \overbrace{\frac{\partial \mathscr{A}_{s+h-1}}{\partial r_s}}^{A\lambda(1+r)\delta \cdot \epsilon^{A,r}} \right) - \overbrace{\lambda_s \mathscr{A}_{s-1}}^{\lambda A} = 0$$

FOCs

- From the r_s derivative around the (unknown) RSS[1]:

$$\lambda^{-1}\epsilon^{U,r} = A - [1 - \delta(1+r)]A\epsilon^{A,r} - \tau N\epsilon^{N,r}.$$

- Same procedure applied to the τ_s derivative:

$$\lambda^{-1}\epsilon^{U,\tau} = (1-\tau)N - [1 - \delta(1+r)]A\epsilon^{A,\tau} - \tau N\epsilon^{N,\tau}.$$

- Eliminating λ_t we eventually obtain $\Rightarrow$?

[1] Here, we omitted δ as an argument of the elasticities.

Optimality Condition

If RSS exists and λ_t converges, it satisfies the government budget constraint and

$$\Big[(1-\delta(1+r))\cdot\ell\cdot(\mathrm{MRS}\cdot\epsilon^{A,r}+\epsilon^{A,\tau})\Big]-\Big[\frac{\tau}{1-\tau}(-\epsilon^{N,r}-\mathrm{MRS}\cdot\epsilon^{N,r})\Big]$$
$$-\Big[\ell\cdot\mathrm{MRS}-1\Big]=0,$$

where $\mathrm{MRS}(\delta)\equiv-\epsilon^{U,\tau}/\epsilon^{U,r}$.

Optimal Condition: Intuition

We decompose the LHS into three items:

(i) $[(1-\delta(1+r)) \cdot \ell \cdot (\mathrm{MRS} \cdot \epsilon^{A,r} + \epsilon^{A,\tau})]$, the **liquidity effect**.

- The planner can raise the present value of resources by issuing more debt when there is a gap between the social discount factor δ and the interest rate.

(ii) $-\left[\frac{\tau}{1-\tau}(-\epsilon^{N,r} - \mathrm{MRS} \cdot \epsilon^{N,r})\right]$, the **labor supply**.

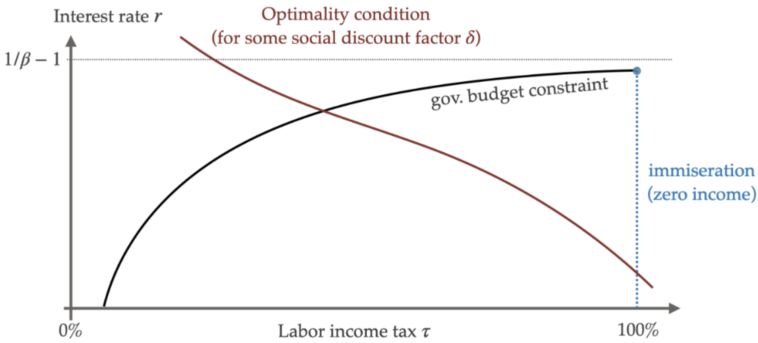
- It captures any disincentive effects on labor supply of greater labor income taxation and interest rates.

(iii) $-(\ell \cdot \mathrm{MRS} - 1)$, the **redistribution**.

- It demonstrates that combined increase in interest rates and labor income taxes has distributional consequences.

Question: Positive or negative?

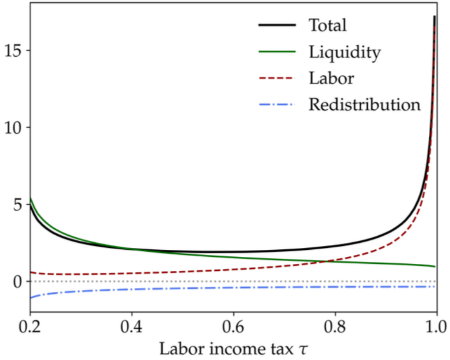
Optimal Condition: Illustration



Parameterization

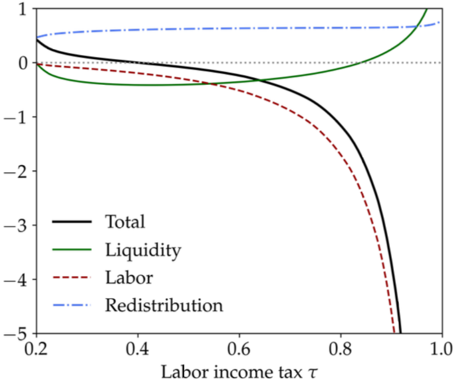
- Standard calibration:
  - AR(1) income process, initial debt $= 100\%$, $G/Y = 20\%$, initial $r = 2\%$.
  - What does the RSS look like? Turns out to depend on the utility function $u(c, n)$.
  - Begin with a log-separable preferences, $u(c, n) = \ln c - v(n)$ with constant Frisch elasticity $= 1$.
  - Later we explore other preferences.

Assume "correct" social discounted factor, $\delta = \beta$.

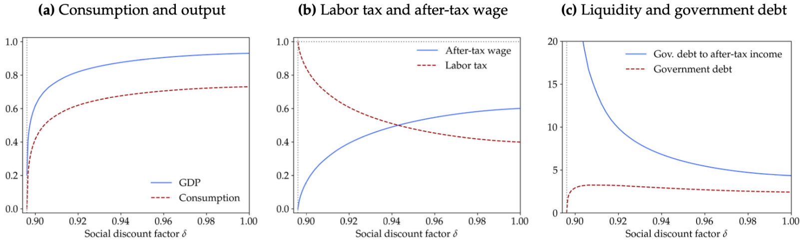


RSS with Various Social Discounted Factor

Same with infinitely patient planner (OSS).



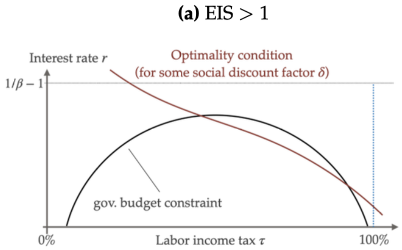
Vary social discounted factor δ between β and 1.



Why no RSS?

benefit-cost balance?

Additively separable preferences $u(c, n)$ with EIS $1/\sigma$ and Frisch elasticity $1/\nu$.



Questions are welcome.